Chapter 2

Basic Theory

Understanding Passive Magnetic Attitude Control (PMAC) begins with an overview of the underlying equations. This chapter serves as a review of the physics governing the components of a PMAC system. First, the basic equation for all rotational motion problems is defined. Next, ferromagnetic theory is reviewed; this is the foundation necessary for the design and study of PMAC systems. This dissertation uses the notation defined in Appedix A.

2.1 Euler's Rotational Equation of Motion

Regardless of attitude parameters used to describe rotational motion, the response of a satellite (or any rigid body) is given by Euler's rotational equations of motion [65]:

$$[I]\dot{\boldsymbol{\omega}} = -[\boldsymbol{\omega}\times][I]\boldsymbol{\omega} + \mathbf{L} \tag{2.1}$$

where $[I]$ is the 3×3 inertia matrix of the rigid body about its center of mass, $\boldsymbol{\omega}$ is the 3×1 body angular velocity vector, $\dot{\boldsymbol{\omega}}$ is the 3×1 derivative of the body angular velocity vector, $\mathbf{L}$ is the 3×1 external torque vector, and $[\boldsymbol{.}\times]$ is the skew-symmetric matrix operator, defined as follows [65]:

$$[\boldsymbol{x}\times] = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}. \tag{2.2}$$

Choosing a body-fixed coordinate system which aligns with the principal body axes results in a diagonal inertia matrix, which simplifies Equation 2.1 as follows [65]:

$$
\begin{aligned}
I_{xx}\dot{\omega}_x &= -(I_{zz} - I_{yy})\omega_y\omega_z + L_x \\
I_{yy}\dot{\omega}_y &= -(I_{xx} - I_{zz})\omega_z\omega_x + L_y \\
I_{zz}\dot{\omega}_z &= -(I_{yy} - I_{xx})\omega_x\omega_y + L_z
\end{aligned}
\tag{2.3}
$$

where the subscript represents the component aligning with a specific principal body axis. Note that Equation 2.3 shows that there will be angular velocity coupling for any non-symmetric rigid body. Equations 2.1and 2.3 are the basis for all of the analytical models and simulations to follow. The difficulty in modeling is in correctly representing the external torque **L** applied to the system.

2.2 Magnetic Theory

2.2.1 Magnetizing Field H vs. Magnetic Flux Density B

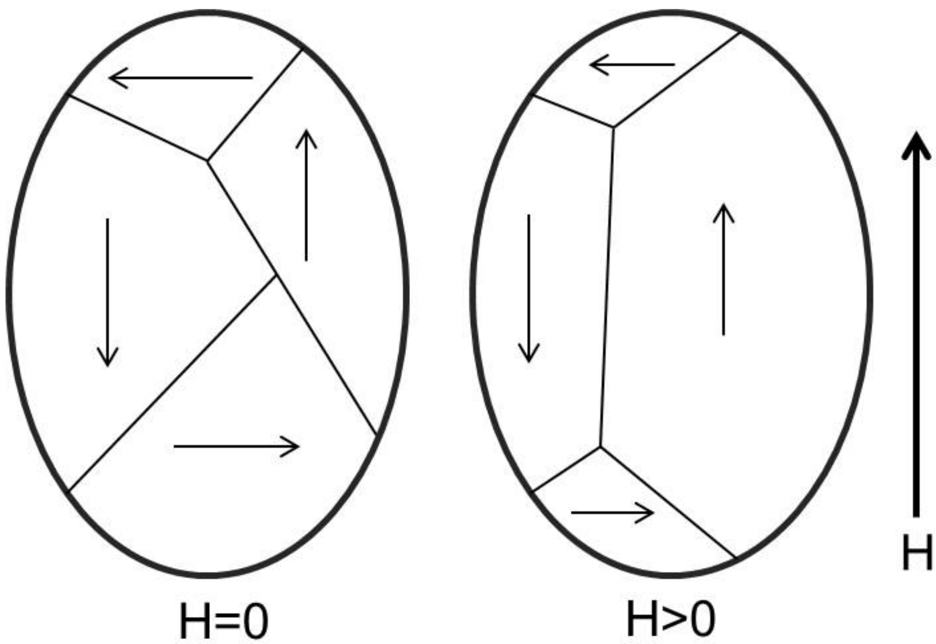
There exist two separate but closely related fields which both, at times, go by the name "magnetic field": the magnetizing field **H** and the magnetic flux density **B**. In the SI system of units, **H** is in units of A/m while **B** is in units of Tesla (T). The relative permeability is defined as $\mu_r = \frac{\mathbf{B}}{\mu_0 \mathbf{H}}$ where the permeability of free space $\mu_0 = 4\pi \cdot 10^{-7}$ T·m/A. For most materials, μ_r is very close to unity, meaning the material does not increase **B** appreciably in response to **H**. However, within ferromagnetic material the situation is much different. The following definition relates **B** and **H**: [17]

$$\mathbf{B} \equiv \mu_0(\mathbf{H} + \mathbf{M}) \tag{2.4}$$

where **M** is the magnetization of the material within which the fields are present. Magnetization is defined as $\mathbf{M} = \mathbf{m}/V$ where **m** is the magnetic moment and V is the magnetized volume; thus the magnetization **M** is the magnetic moment **m** density. For materials with μ_r close to unity (such as air), **M** is very close to zero, and Equation 2.4 reduces to $\mathbf{B} = \mu_0\mathbf{H}$.

However, a ferromagnetic material has a non-zero magnetization which changes in response to an applied field. This change in **M** is due to microscopic changes within the material. Figure 2.1

Figure 2.1: Example magnetic domains are shown. With zero applied field (left), the domain orientation of the magnetic material is such that the sum magnetization of the material is small. However, when a magnetizing field is applied to the material (right), the domains oriented parallel to that field grow as the out-of-alignment domains shrink.



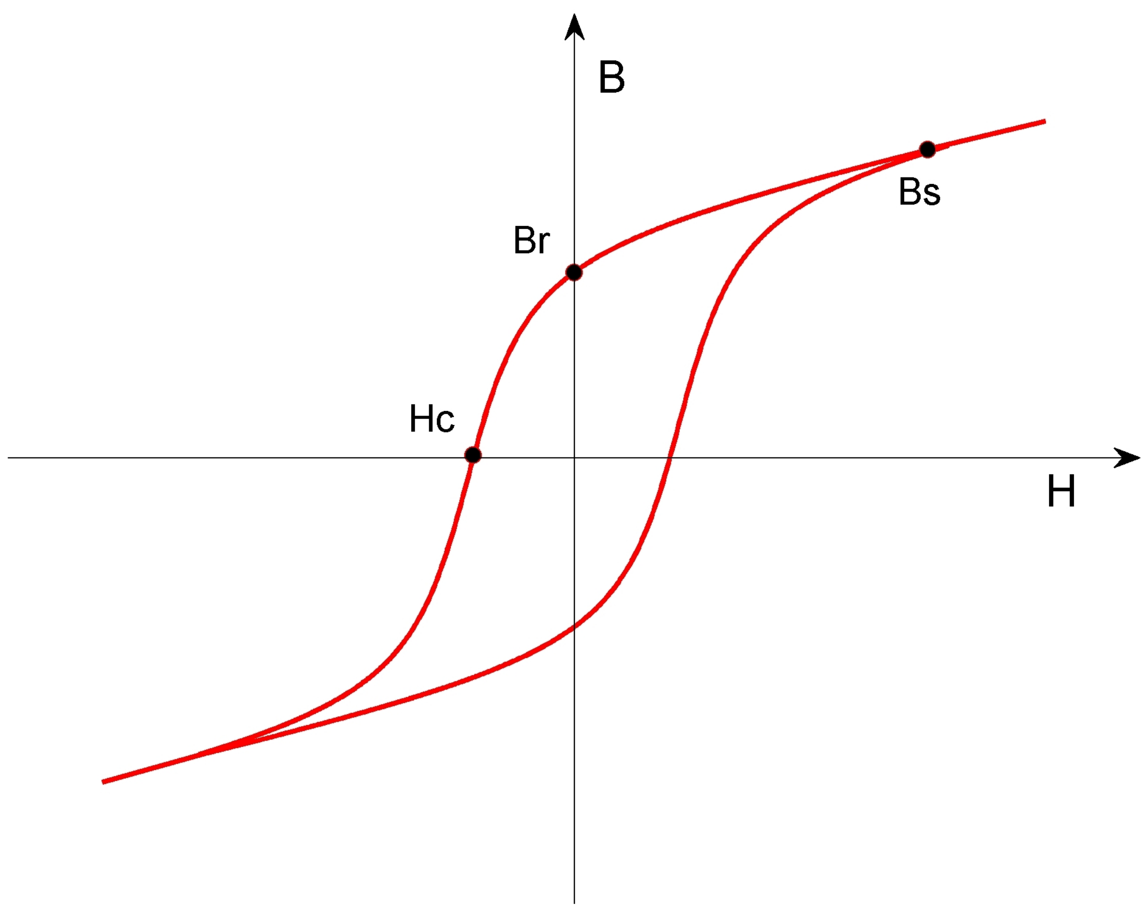
is an example of multiple microscopic magnetic domains within a material. Each magnetic domain is composed of a group of atoms with parallel magnetic moments. When zero magnetizing field is present, the sum magnetization of the material is small because the magnetic moments of multiple domains are mostly canceled. However, when a magnetizing field is applied, a domain with a magnetic moment parallel to the field will grow as the magnetic moments of atoms close to its boundaries align to the applied field. This change in total magnetization **M** is non-linear with changes in **H**, and is responsible for the familiar hysteresis loop [8].

2.2.2 Hysteresis Loops

When a ferromagnetic material is subjected to a changing magnetizing field H and the magnetic flux density B is measured, plotting B vs. H will result in a hysteresis loop such as the one shown in Figure 2.2. A major hysteresis loop may be defined by three parameters: the coercivity H_c, the remanence B_r, and the saturation B_s. The coercivity is the applied field necessary to bring the B field to zero, or the x-axis intercept. The remanence B_r is the remaining B within the material when H has been decreased to zero, or the y-axis intercept. There exists a maximum value of M for a given material. If the applied field H is subtracted from B, the curve $B/\mu_0 - H$ vs. H will asymptotically approach this maximum magnetization, known as saturation [10]. At saturation (and only at saturation), the magnetization M within a bar or cylinder sample is constant and uniform. This is because all of the individual magnetic domains within the material have aligned in the same direction. The material cannot supply any more magnetization because there are no more domains to align. Thus, after saturation, the increase in B-field is solely due to the increase in magnetizing field, and thus has a slope of μ_0. The point on the hysteresis curve at which B_s starts to increase with slope μ_0 is the saturation flux density. The area enclosed within the hysteresis loop is an important feature; it represents the energy absorbed by the magnetic material per unit volume as it completes one magnetization cycle.

A hysteresis loop may be split into a lower and upper curve which are generated by the increasing and decreasing sections of magnetizing field cycling, respectively. The hysteresis fitting

Figure 2.2: An example magnetic flux density B vs. magnetizing field H hysteresis loop. The coercivity H_c, remanence B_r, and saturation B_s are shown. The area encircled by the hysteresis loop is the energy loss per cycle per unit volume. After saturation B_s, the slope of the hysteresis loop is simply μ_0.



described in Section 7.3 relies upon this bifurcation. The upper and lower curves of the hysteresis loop will be odd-symmetric if the cycle amplitude remains constant and there is no DC offset in the magnetizing field. This property will also be used in the hysteresis fitting.

2.2.3 Magnetic Property Dependencies

The shape of a magnetic hysteresis loop depends on many things, but some of the more important factors include: material composition, degree of heat treatment, applied H-field extrema, applied field offsets, frequency of H-field cycling, and sample dimensions. The material itself governs the saturation magnetization amplitude (the applied field at which saturation occurs and the shape of the curve from 0 A/m to saturation are not determined solely by the material as they are structure-specific); the inherent crystal structure of the material composition defines "easy" directions of magnetization [17]. Note that most nickel-iron alloys (such as HyMu-80) have low magnetocrystalline anisotropy after heat treatment, meaning the ease of magnetization is about the same regardless of direction [5].

Heat treatment can restore the crystalline structure of a material that is damaged during cold work, such as extruding, rolling or bending. Heat treatment also serves to break down the walls between magnetic domains, increasing the mean domain size and allowing the magnetic material to be magnetized to higher levels at lower magnetizing fields. Hysteresis loops before and after heat treatment will likely be very different.

The magnitude of the applied field cycle will change the resultant hysteresis loop [17]; this effect is included within the Flatley hysteresis model (described in Section 8.1.6.5). Figure 2.3 shows the output of the Flatley hysteresis model for cycle amplitudes of ± 2 A/m, ± 3 A/m, and ± 8 A/m. Note that the loop area decreases substantially as the applied field decreases. A constant offset in the applied field can also distort the hysteresis loop. In the case of a satellite PMAC system, such a constant offset may be provided by proximity to current loops or a bar magnet. As shown in Figure 2.4, an H-field offset pushes the hysteresis loop away from the origin. If the cycle amplitude approaches the material saturation, this magnetizing field offset can result in a smaller loop area, and thus, decreased dampening.

The frequency of applied field cycling can effect the hysteresis loop measured. Figure 2.5 shows how an increase in cycle frequency tends to increase the coercivity H_c. More energy is used to switch the magnetic domains at higher cycle frequencies. However, a DC hysteresis curve (usually defined as an applied field cycle frequency of 10 Hz or less) is minimally affected by frequency. All hysteresis loops measured in Section 7.3 are produced by applying a field with a cycle frequency of less than 1 Hz. The hysteresis loop is also affected by the demagnetizing field of the test sample, which is further examined in the following section.

2.2.4 Demagnetizing Fields

Anyone who has handled magnets is familiar with the idea of a magnetic pole. Consider a bar magnet that has been magnetized by a magnetizing field H in the left to right axial direction. After H has been removed, there exist two magnetic poles, the south pole on the left and north pole on the right. Figure 2.6 shows the H and B fields resultant of the poles. As shown, there exists an H field outside and inside the magnet. Outside the magnet, the simple relation $B = \mu_0 H$ holds. However, inside the magnet, an H field opposes the B field and is termed the demagnetization field H_d. Equation 2.4 becomes $B = \mu_0(-H_d + M)$. If an external applied field H_a is present, Equation 2.4 becomes:

$$B = \mu_0(H_a - H_d + M). \tag{2.5}$$

Demagnetizing fields are difficult to calculate, but are directly proportional to the magnetization of the bar magnet: $H_d = N_d M$ where N_d is the demagnetization factor. The demagnetization factor varies mainly as a function of the length to diameter ratio L/D of the sample, but also varies as a function of magnetization: different N_d values are used at values close or far from saturation. These limitations make it especially complicated to calculate the hysteresis loop of a material; empirical determination is much more accurate.

One way to measure the hysteresis loop of a material without having to account for demagnetizing fields is to use a toroid-shaped sample. Lines of magnetic flux density B, which follow closed

Figure 2.3: Example B vs. H hysteresis loops for various applied field cycle magnitudes. Figure was generated using the Flatley hysteresis model and the HyMu-80 closed magnetic circuit hysteresis parameters (see Table 7.2).

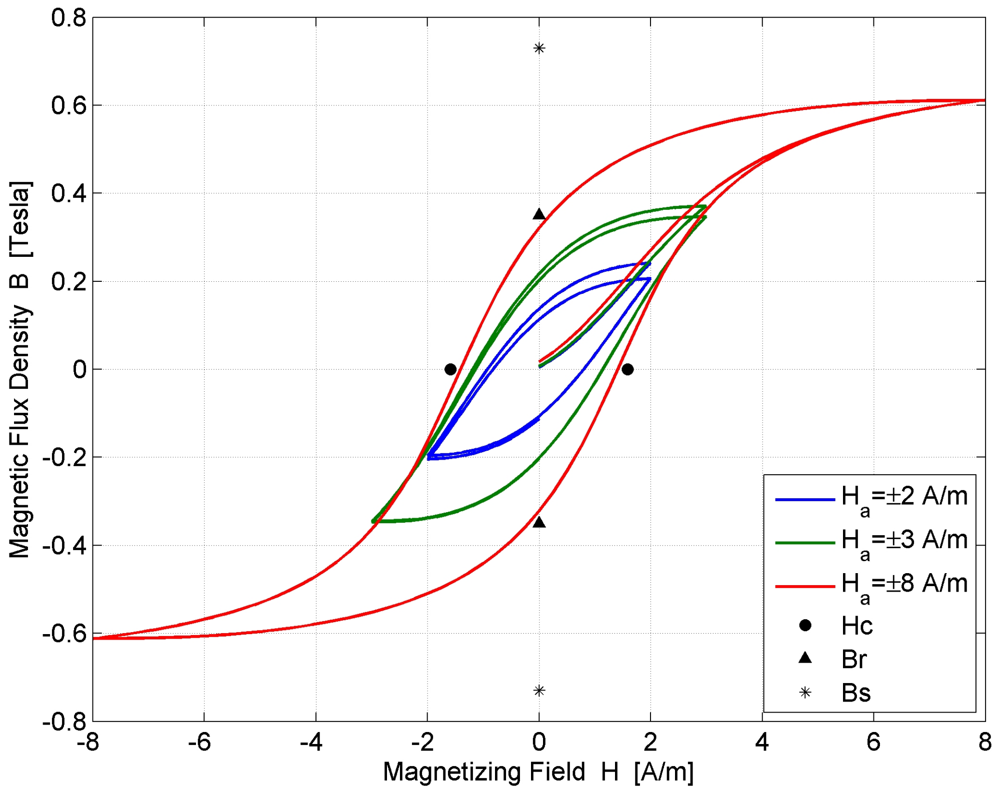


Figure 2.4: Example B vs. H hysteresis loops for various applied field DC offsets. Figure was generated using the Flatley hysteresis model and the HyMu-80 closed magnetic circuit hysteresis parameters (see Table 7.2). All three datasets are generated using an AC magnetizing field cycle amplitude of ± 2 A/m.

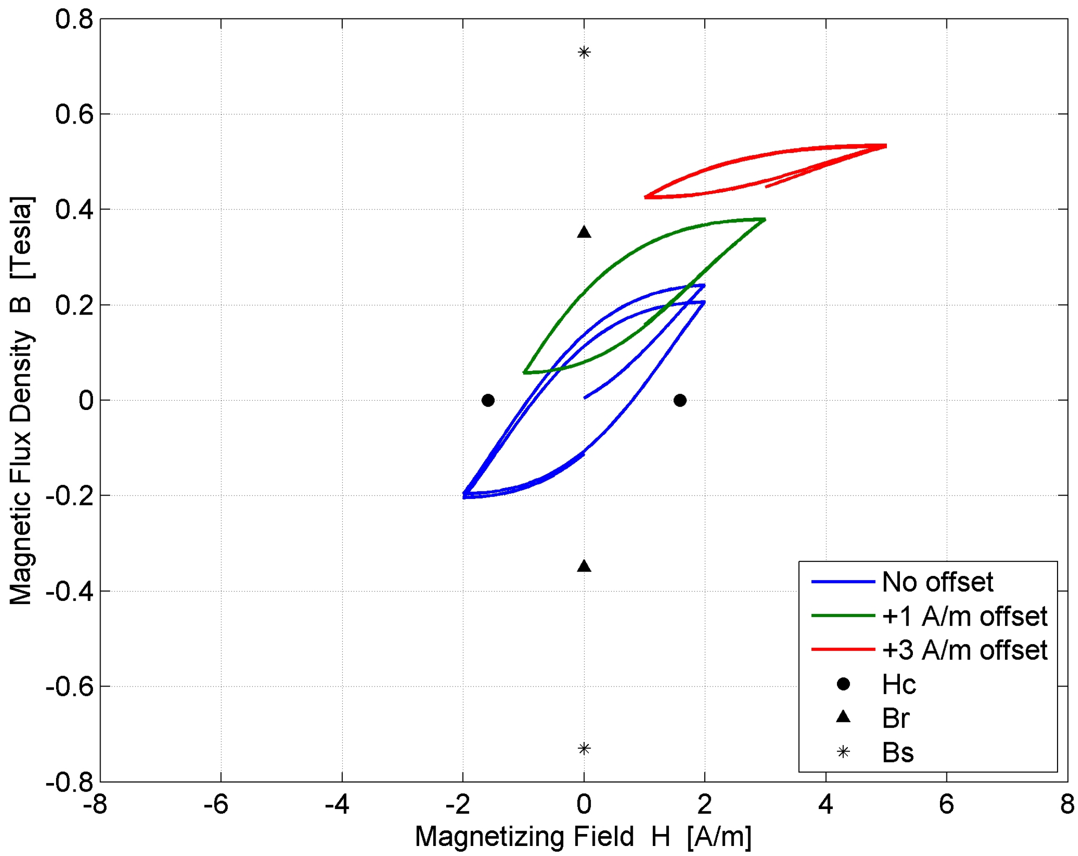


Figure 2.5: The effect of applied field cycle frequency on the hysteresis loop. The coercivity H_c and the loop area increase as the hysteresis loop is cycled at increasing frequency. Image used from [8].

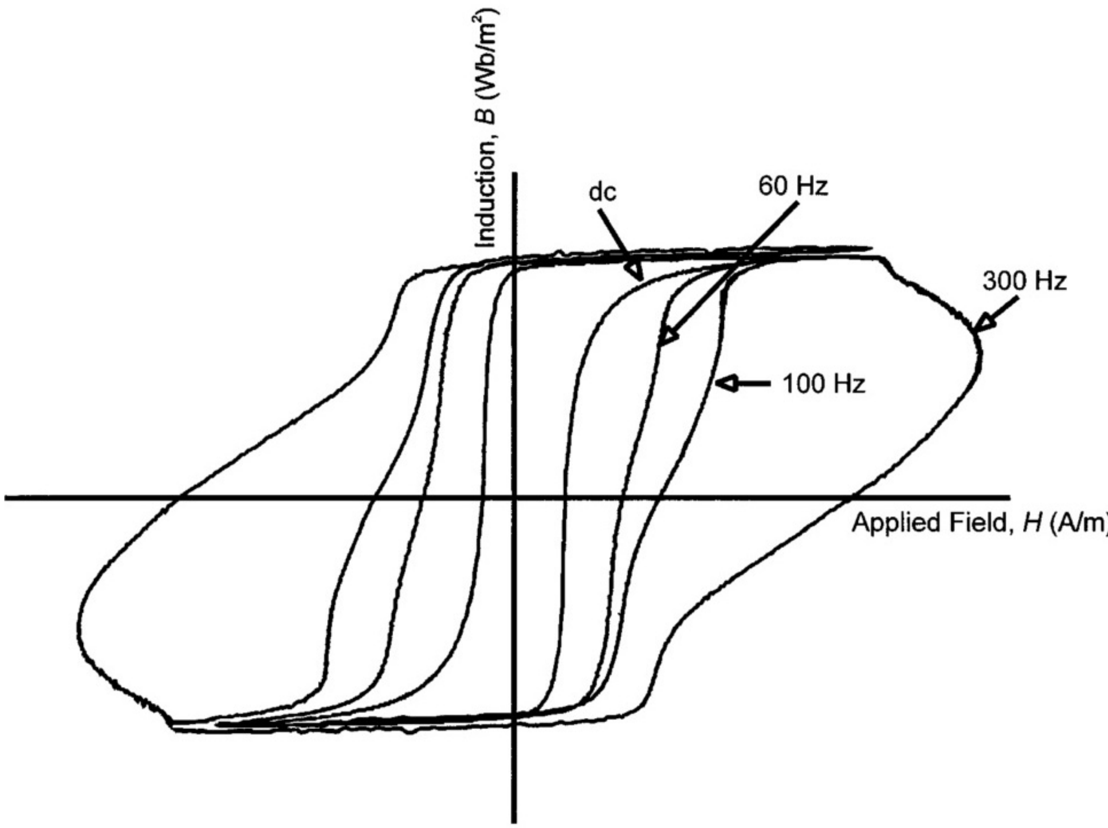
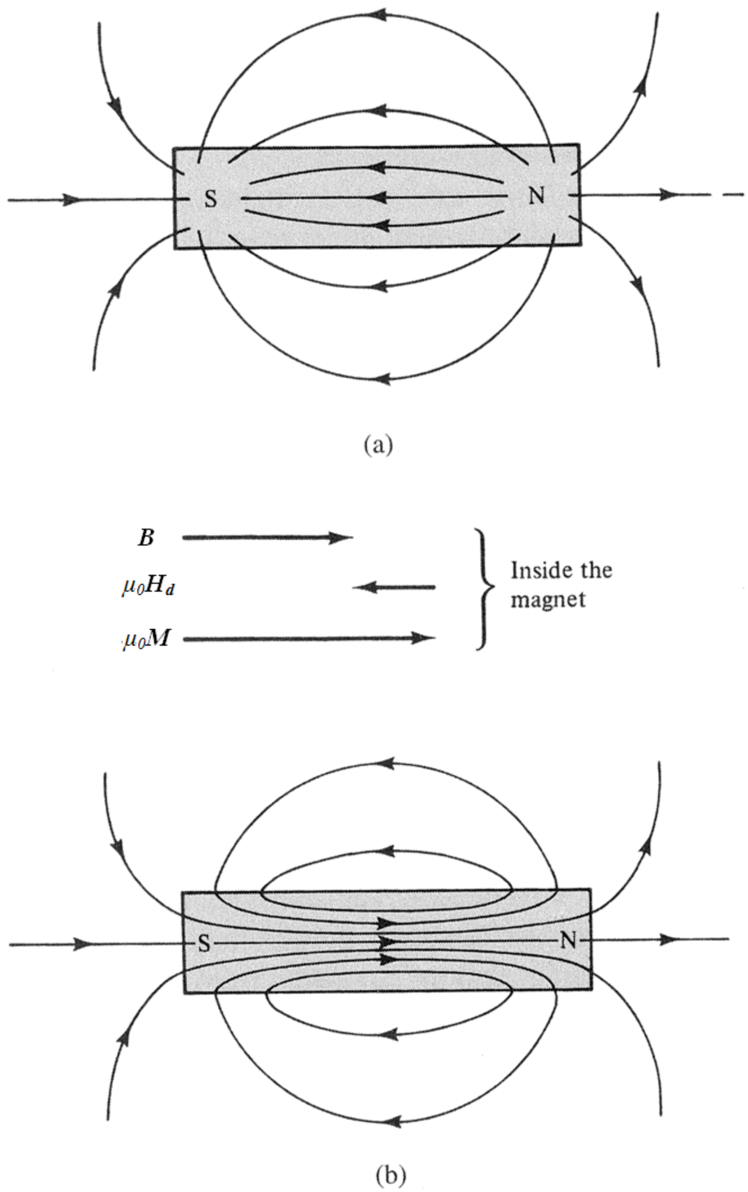


Figure 2.6: (a) **H**-Field and (b) **B**-Field of a bar magnet when there is zero applied field. Note that $\mathbf{M} > 0$ only within the magnet, and that outside the magnet, $\mathbf{B} = \mu_0\mathbf{H}$. Figure adapted from [17].



loops, lie entirely within such a sample. This type of sample is known as a closed magnetic circuit. This means that, even when magnetized, no poles are present in such a sample, and thus no demagnetizing field H_d is present in the sample. The hysteresis loop measured from a toroidal sample would be B vs. H_{true}. The true magnetic field within the material is a combination of the applied magnetizing field field and the demagnetizing field generated by the material: $H_{\text{true}} = H_a - H_d$. Of course, if $H_d = 0$ (as in a toroidal sample), then $H_{\text{true}} = H_a$. However, this is not the case for open magnetic circuit samples. Because the closed magnetic circuit hysteresis loop B vs. H_{true} is invariant with respect to the dimensions of the material, this hysteresis loop is generally what is quoted on many material data sheets.

However, the closed magnetic circuit hysteresis loop is not a good representation of the true open magnetic circuit hysteresis loop for the rods or strips typically used in a PMAC system. Testing to date has shown that the loop areas loops differ by one to two orders of magnitude [63]. As the loop area is directly related to the dampening provided by each hysteresis rod, this has vast implications for a numerical simulation of the attitude dynamics. Section 7.3 presents measurements of the true open magnetic circuit loops for hysteresis rods which are typically used in PMAC systems.

2.2.5 Magnetic Torques

All magnetic torques obey the following formula:

$$\boldsymbol{L} = \boldsymbol{m} \times \boldsymbol{B} \tag{2.6}$$

where **m** is the magnetic moment vector and **B** is the local magnetic flux density vector. In most situations, **B** is due to earth's local magnetic field alone. Thus, the torque is based on the value of **m** for various magnetic materials. The high coercivity of a permanent magnet prevents the earth field from changing its magnetization, thus for a bar magnet, **m** is constant and may be measured (see Section 7.2). However, determining the magnetic moment of a hysteresis rod is more difficult.

2.2.6 Hysteresis Rods

A PMAC system necessarily uses bar- or cylinder-shaped hysteresis material. This is required because such samples are magnetized mainly in the axial direction, which produces a torque as defined by Equation 2.6. For these open magnetic samples the demagnetizing field H_d is unavoidable, and the L/D ratio is limited by the dimensions of the spacecraft and the necessary volume of hysteresis material. This means that B measured for an open magnetic circuit (with demagnetization) will be significantly less than B measured for a closed magnetic circuit.

As shown in Equation 2.6, the component of $\mathbf{B}$ parallel to the magnetic moment $\mathbf{m}$ does not produce a torque. Assuming the majority of uniform magnetization is parallel to each rod's axis, the rod produces a negligible B-field perpendicular to its axial direction. Thus, sets of rods which are co-planar but have some separation should have minimal interaction. The general rule of thumb for hysteresis rod placement is that the perpendicular distance between two rods should be greater than 30-40% of their length [57]. Given this separation, the assumption is made that the interaction of multiple hysteresis rods may be ignored in analysis. Thus, the magnetic flux density $\mathbf{B}$ on the right side of Equation 2.6 is generated solely by the local earth B-field. This means that the magnetic moment $\mathbf{m}$ is the only characteristic of the rod which contributes to the torque produced by the rod.

Therefore, in order to model the torque due to the hysteresis rods, the magnetic moment $\mathbf{m}$ of the rod must be defined. First, it is assumed that the magnetic moment is entirely parallel to the rod. Using $m = M/V$, $H_d = -N_d M$ and Equation 2.5, the magnetic moment parallel to the rod for an open magnetic circuit may be defined as [17]:

$$m = V\left(\frac{B/\mu_0 - H_a}{1 - N_d}\right) \tag{2.7}$$

where B is the average parallel magnetic flux density within the rod and H_a is the applied field parallel to the rod. Many groups ([47],[61],[58],[63],[26]) simplify Equation 2.7 to $m = VB/\mu_0$. It is feasible to ignore N_d in the denominator for rods with $L/D > 30$ [17], as the error is $< 2\%$ (the demagnetizing field has not been ignored, it is taken into account by emperically measuring B).

However, the hysteresis curve must be measured to ensure H_a is negligible with respect to B/μ_0. None of the groups reviewed in Section 3.3 state their assumptions in using the simplified formula. Some make the grave mistake of assumming B within Equation 2.7 is given by a material data sheet (usually B vs. H_{true}). This assumption saves one from having to measure the hysteresis loop, but also introduces serious errors because it ignores the effect of the demagnetizing field. The B used within Equation 2.7 is the average interior magnetic flux density (across the length of the sample), and must be measured for the open magnetic circuit to produce an accurate magnetic moment m. The B vs. H_a hysteresis loop is used in the PMAC dynamics simulation (see Section 8) because it directly relates B to the earth H-field encountered by the spacecraft, which is H_a.