Chapter 5

Control System Design

We anticipate that satellite developers will refer to this dissertation when designing future PMAC systems. One example of good PMAC system design is the Colorado Student Space Weather Experiment (CSSWE) CubeSat (discussed in Chapter 4). This chapter details PMAC system design practices using CSSWE as a specific example.

The attitude control system of CSSWE has two performance requirements:

(1) The attitude control system shall have a settling time of less than 7 days.

(2) Once settled, the attitude shall stay aligned within 15° of the local magnetic field.

5.1 Maximum Expected Environmental Torques

A successful attitude system design begins with an analysis of maximum expected spacecraft environmental torques. Table 5.1 shows the expected environmental torques in the CSSWE environment. Methods to calculate these torques are explained in Section 8.1.6. The maximum expected non-magnetic torque total is used to determine the minimum acceptable bar magnet magnetic moment.

Table 5.1: Worst-case environmental torque magnitudes for the CSSWE 3U CubeSat. This analysis assumes a 3U CubeSat in a 480km×790km, 65° orbit and moderate solar input.

Torque	Maximum Value [N·m]
Magnetic Residual $\|\|\mathbf{L}_{\mathrm{R}}\|\|$	$4.3E-7$
Aerodynamic $\|\|\mathbf{L}_{\mathrm{D}}\|\|$	$1.8E-8$
Gravity Gradient $\|\|\mathbf{L}_{\mathrm{G}}\|\|$	$3.2E-8$
Solar Pressure $\|\|\mathbf{L}_{\mathrm{SP}}\|\|$	$2.5E-9$
Sum $\|\|\mathbf{L}\|\|_{\mathrm{sum}}$ (excluding $\|\|\mathbf{L}_{\mathrm{R}}\|\|$)	$5.3E-8$

5.2 Bar Magnet Design

In the CSSWE orbit, $||\mathbf{B}||$ varies from 18 to 52 μTesla. We present a modified version of Santoni and Zelli's [63] minimum recommended bar magnet strength:

$$m_{\mathrm{min}} = 15 \left(\frac{||\mathbf{L}_{\mathrm{sum}}||}{||\mathbf{B}||_{\mathrm{min}} \cdot \sin(\beta_{\mathrm{max}})} \right) \tag{5.1}$$

where $||\mathbf{L}||_{\mathrm{sum}}$ is the sum of the independent, non-magnetic environmental torque magnitudes, $||\mathbf{B}||_{\mathrm{min}}$ is the minimum magnetic flux density magnitude experienced by the satellite, and β_{max} is the desired pointing accuracy. Although the required alignment with the magnetic field is 15°, the system is designed using $\beta_{\mathrm{max}} = 10°$ to ensure there is adequate margin in the PMAC system design. Santoni and Zelli's [63] version of Equation 5.1 defines $||\mathbf{L}||_{\mathrm{sum}}$ as the sum of all independent environmental torques. However, $||\mathbf{L}||_{\mathrm{sum}}$ is better defined as the sum of the non-magnetic torques. Instead of heavily weighting Equation 5.1 due to the maximum magnetic residual torque vs. the minimum bar magnet torque, the respective magnetic moments may be compared directly. Because any magnetic torque is given by Equation 2.6, as long as $m_{\mathrm{bar}} \geq 15 m_{\mathrm{res}}$, the magnet will easily overpower the magnetic residual. This revised definition of $||\mathbf{L}||_{\mathrm{sum}}$ produces a less extreme bar magnet moment, thereby reducing the necessary hysteresis damping material within the volume-limited CubeSat and lessening the initial magnetic potential energy of the satellite.

The bar magnet moment is also directly related to the initial energy which may be introduced to the system when a small satellite is released from its launcher. The rotational energy of a PMAC satellite may be split into the kinetic and potential energy as shown below:

$$T_{\mathrm{K}} = \frac{1}{2}\boldsymbol{\omega}^T[I]\boldsymbol{\omega} \tag{5.2}$$

$$T_{\mathrm{P}} = -\mathbf{m} \cdot \mathbf{B} \tag{5.3}$$

where $\mathbf{m}$ is the total satellite magnetic moment, $\boldsymbol{\omega}$ is the angular velocity vector, and $[I]$ is the satellite moment of inertia matrix. Usually, small satellites are secondary payloads and thus cannot control the initial angle versus the magnetic field β_0 at satellite deployment. Consider a satellite which initially posseses zero kinetic energy and starts with $\beta_0 = 180°$. In this case, the initial rotational energy is directly related to the magnetic moment magnitude $||\mathbf{m}||$. Thus, the bar magnet moment must balance between dominating the disturbance torques and limiting the possible system energy which could be introduced at deployment.

For the considered conditions, Equation 5.1 yields an ideal bar magnet strength $m_{\mathrm{min}} = 0.25$ A·m^2, which is 25× the expected residual magnetic moment. A bar magnet was ordered to meet this level, but when the bar magnet was measured (see Section 7.2), it was found to be $m_{\mathrm{bar}} = 0.80 \pm 0.017$ A·m^2. Due to limited time time before CubeSat delivery and because the measured $m_{\mathrm{bar}} > m_{\mathrm{min}} > 15m_{\mathrm{res}}$, the bar magnet magnetic moment was deemed acceptable and is used in the CSSWE PMAC system.

5.3 Hysteresis Rod Design

Once a bar magnet dipole moment has been chosen, the hysteresis rod dimensions and quantity should be determined. Usually in a PMAC system, hysteresis rods are mounted in pairs orthogonal to the bar magnet to reduce the offset of the applied field due to the bar magnet (see Section 2.2.3). Thus, the CSSWE bar magnet is aligned with the minor inertia axis (${}^{\mathcal{B}}Z$) of the CubeSat and the hysteresis rods are mounted on orthogonal axes (${}^{\mathcal{B}}X$ and ${}^{\mathcal{B}}Y$, see Figure 4.3). The design below assumes that perpendicular rods never interact and that rod sets do not interact when separated by more than 30% of their length [57].

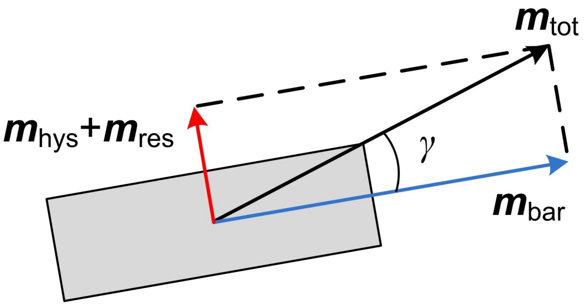
As explained in Section 2.2.6, the length to diameter ratio of a hysteresis rod has a large effect on its performance. Higher length to diameter ratios result in higher permeability hysteresis rods. The maximum length of the hysteresis rod is limited by the dimensions of the spacecraft. For CSSWE, the interior spacecraft dimensions perpendicular to the bar magnet limit the rods to 9.5 cm in length. Typical values of the length to diameter ratios for hysteresis rods are on the order of 100 [57]. Thus, CSSWE used hysteresis rods of length 95 mm and diameter 1 mm.

Next comes the question of how many pairs of hysteresis rods to include in the system. First, there is a volume limitation; the planes of orthogonal rod sets must be separated by at least 30% of the length of one rod in order to ensure that the magnetization of one hysteresis rod set does not affect the other [57]. Also, it may be advantageous to separate the hysteresis rods from any spacecraft magnetometers. It is possible to calibrate a magnetometer to remove hysteresis effects, but most calibration methods assume a linear hysteresis curve [28] [72]; the magnetometer performance may degrade if the hysteresis rods are too close.

Regardless of the physical limitations of including hysteresis in the system, the optimal amount of hysteresis material is a question. If not enough hysteresis material is included, the system will take too long to converge, and PMAC design rule #1 will be violated. As dampening material is increased, the offset from the local magnetic field increases as well, because the total magnetic moment vector is what will align with the local magnetic field. Figure 5.1 defines the error angle γ between the total magnetic moment vector and the magnetic moment vector of the bar magnet. If the sum of the maximum hysteresis magnetization and the magnetic residual moment represent a significant fraction of the total magnetic moment vector, the error angle γ may cause the system to violate PMAC design rule #2.

Of course, the available locations within the satellite to affix hysteresis rods also determines the allowable number of rods. Figure 5.2 shows a solid model of the final design of the CSSWE PMAC system (the bar magnet and hysteresis rods are highlighted in red). CSSWE uses three hysteresis rods on each of the ${}^{\mathcal{B}}X$ and ${}^{\mathcal{B}}Y$-axes. The rods are separated by a perpendicular distance

Figure 5.1: There exists an error angle γ due to magnetization on the satellite not parallel to the bar magnet.



of at least 3.25cm (34% of their length) and the bar magnet to ensure the hysteresis rods have minimal magnetic offset. The magnetometer (not shown) is located on an electronics board near the bottom of the satellite. The magnetometer was chosen to be separated from the hysteresis rods rather than the bar magnet because a constant offset from the bar magnet can be negated via calibration, but the non-linear variation of magnetic fields due to the hysteresis rods are more difficult to remove.

Accurate magnetic moment and hysteresis parameters input to a realistic PMAC simulation can be used to determine the volume of spacecraft hysteresis material needed to meet specific mission requirements. With the necessary hysteresis volume defined, multiple rods may be installed using the design rules outlined above. Thus, an accurate attitude dynamics simulation is a key part of PMAC system design. Chapter 7 ensures accurate input to this simulation through hysteresis rod measurement; it also tests the hysteresis design rules outlined above. Chapter 8 defines and tests a PMAC attitude dynamics simulation. This simulation is verified for future design use by comparing its output with the on-orbit attitude data from the CSSWE CubeSat. As such, Chapter 6 develops a filter for attitude determination of a PMAC satellite and applies it to the empirical CSSWE data.

Figure 5.2: A solid model highlighting the position of PMAC components. The hysteresis rods (top) have a large separation from the bar magnet (bottom left) in order to prevent magnetic offsets to the rod hysteresis loops. Each hysteresis rod position is labeled; the rod sets are separated by a minimum perpendicular distance of 3.25 cm.