Chapter 3

Background

Although difficult to model due to the nonlinear behavior of hysteresis, Passive Magnetic Attitude Control (PMAC) is simple to realize on a spacecraft: it only requires the installation of a bar magnet and a few hysteresis rods. Thus, early satellites made considerable use of PMAC. Small satellites echo this trend today as electronic components shrink faster than attitude control actuators. The previous PMAC analysis is traced through the mission history of PMAC satellites as well as the analytic and numeric models which have been developed in parallel with these missions. A review of PMAC hysteresis rod measurement to date is presented at the end of the chapter.

We find no previous work which verifies the performance of a PMAC attitude dynamics simulation through comparison to on-orbit attitude data from a PMAC satellite. Neither do we find previous work which measures hysteresis rod performance as affected by the system-level PMAC component magnetization. There is much to be learned in investigating these avenues.

3.1 Mission History

It is not surprising that small satellites today echo the development of large satellites in the early space race. Although satellite electronics shrink very quickly, it is hard for precision actuation devices to keep pace. PMAC is in use today just as it was when it was designed in the 1960's. Studying the missions which have used PMAC sheds light on the type of missions for which it is best suited, and tells the story of its development over time.

3.1.1 Early History of Passive Magnetic Attitude Control

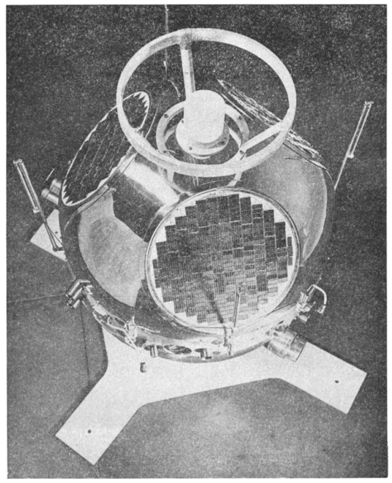
Passive attitude control systems were used in early spacecraft because software and actuation hardware were not yet developed. As spacecraft developed, simple attitude control systems such as spin-stabilization gave way to more complicated (yet still passive) methods, such as gravity-gradient and passive magnetic attitude control.

Passive Magnetic Attitude Control (PMAC) was first used in space in April of 1960 [20]. Researchers at Johns Hopkins Applied Physics Laboratory came up with the method for the Navy's Transit experimental satellite program. The first satellite to use a passive magnetic attitude control system was Transit 1B (Transit 1A did not achieve orbit due to a launch failure); it used PMAC to point the spacecraft toward ground stations in the northern hemisphere. Transit 1B was a spherical satellite with a 10 $A{\cdot}m^2$ bar magnet and two sets of permeable rods, both arranged arranged in a crosshatch pattern. Both sets of permeable rods had 4 rods each, with the second set rotated 45° from the first set, yet still within a plane perpendicular to the bar magnet. The satellite had an initial spin of 17.5 rad/s, which was reduced to 16.3 rad/s after 7 days (this decrease is likely due to the PMAC system). A mechanical de-spin via release of weights from the spin axis reduced the spin rate to 0.5 rad/s. The satellite spin rate decreased to 0.03 rad/s (1.8 deg/s) after another 7 days. The Transit 1B PMAC system was successful enough for Transit 2A to be launched without the mechanical de-spin included; it relied solely on the PMAC system. Transit 2A decreased from 5.0 rad/s to 0.13 rad/s (7.2 deg/s) within 24 days [25]. The analytical model used to analyze the PMAC performance of both Transit satellites is presented in Section 3.2.1.

Due to the success of the Transit 1B and 2A, PMAC was used for Injun 1, the first satellite built entirely by a university [27]. Launched in 1961, Injun 1 failed to separate from GREB, another satellite that was on the same launch [9]. A later satellite from the University of Iowa under the supervision of Dr. Van Allen, Injun 3, was launched in 1963 into a 237 × 2785 km, 70.4° inclination orbit. Its PMAC system aligned to an average deviation of less than 10° from the magnetic field line after a period of 2 weeks [29].

Figure 3.1: The Transit 1B Satellite, the first satellite to use Passive Magnetic Attitude Stabilization. [1]

Figure 3.2: The Injun 3 Satellite, an early university satellite which successful aligned to the magnetic field using PMAC. [56]



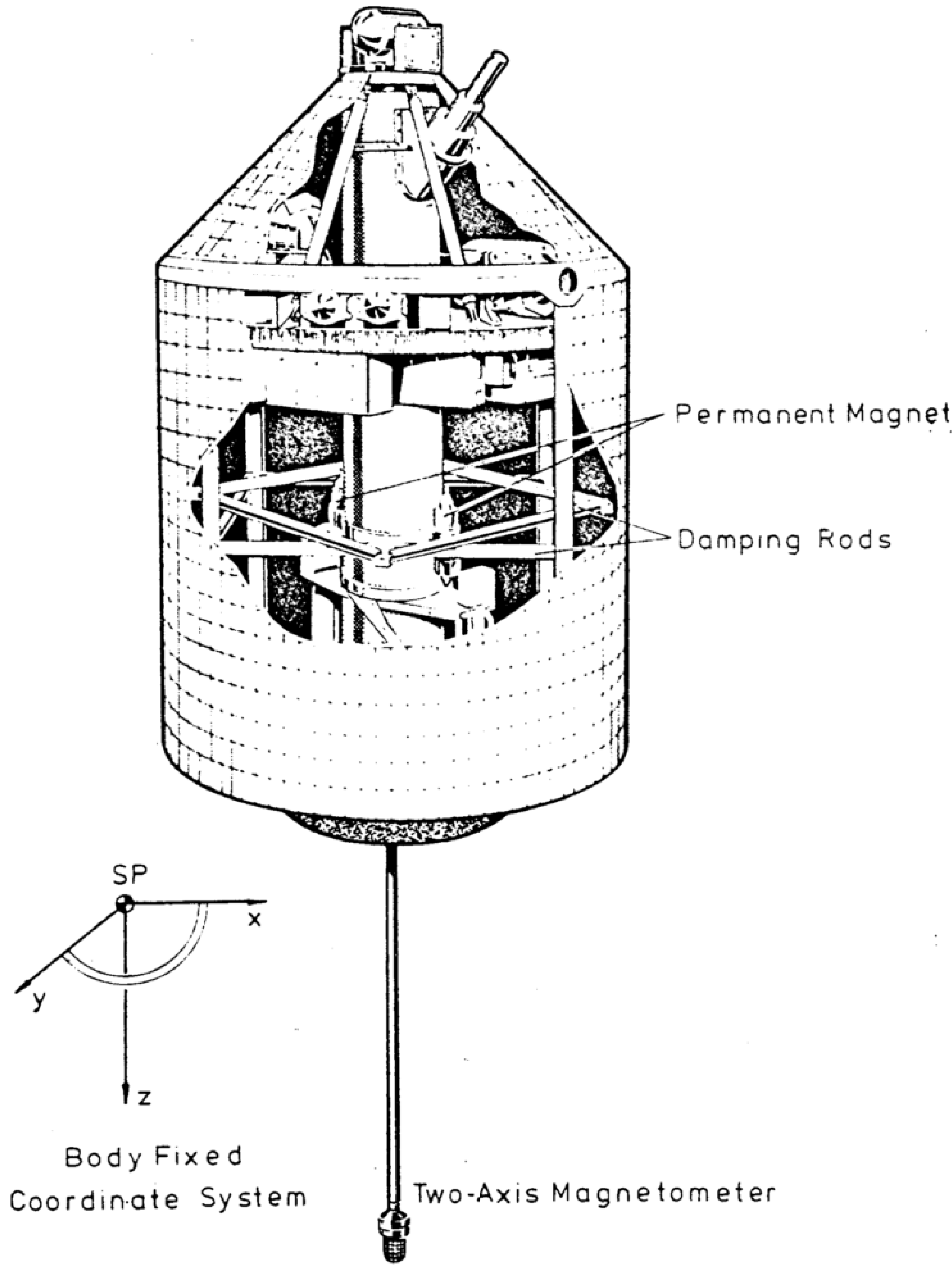
The first German satellite Azur was launched in 1969 and carried a 97 A·m^2 bar magnet. Azur damped from an initial spin rate of 1.25 rad/s down to 0.01 rad/s (0.6 deg/s) and a magnetic field offset of less than 15° within 2.25 days [53].

This section is not an exhaustive list. Other early satellites using PMAC include Transit 2A (1960), ESRO 1A (1968), ESRO 1B (1969), Exos (1978) and Magion (1978) [57]. By the mid 1970's, analysis of PMAC was mostly comprised of analytical models. The use of PMAC begins to wane at this point, as active control systems enable specialized pointing methods. The difficulty of accurately predicting PMAC performance was likely a driver of its decreased use.

3.1.2 Modern Use of Passive Magnetic Attitude Control

As computers and actuators decrease in size, the satellite community has grown increasingly interested in small satellites due to their low launch costs and simplicity. While technology is in development to control these satellites using small actuators, the small satellite community has witnessed a return to passive methods. These methods are especially popular among student missions or technology demonstrations where component price and complexity join size as major

Figure 3.3: Azur, the first German satellite, used passive magnetic attitude control [53].



design factors. PMAC is also popular among science missions which benefit from alignment with the local magnetic field.

The 6kg Swedish *Munin* satellite contained a PMAC system. During design, the satellite used the following requirement: align to within 15° of the earth's magnetic field lines within three weeks. Ovchinnikov developed a numerical attitude simulation and predicted that in order to meet the setting time requirement, the initial angular velocity had to remain less than 10.5 deg/s on each axis [57]. The satellite launched in November 2000 but contact was lost in February 2001. Johnsson indicates that results from the attitude determination analysis are questionable [38]. Attitude determination analysis from *Munin* has not been released to date. Thus, the Ovchinnikov attitude simulation has not been validated; details of the simulation are described in Section 3.3.2.

UNISAT-3 was designed and built by students at the University of Rome. It was launched into a 710km x 790km, 98° inclination orbit on June 29, 2005, and used a PMAC system. The PMAC system used a permanent magnetic with magnetic moment 1 A·m^2 and one hysteresis rod per axis with dimensions 15cm×0.1cm. The only dedicated attitude measurement device was a 3-axis magnetometer. The magnetometer and solar panel currents were used to piece together its three-axis attitude. The Z-axis of the magnetometer did not function on-orbit, so the total magnetic field was estimated using the spacecraft position and the IGRF magnetic field model [62]. UNISAT-3 oscillated about the earth magnetic field at an amplitude of about 30°. The team believed this response was due to under performing hysteresis rods [63].

UNISAT-4 was next in the series of educational satellites. Researchers at the University of Rome determined that the magnetic properties of the hysteresis rods must be measured, as specific dimensions and orientations of the rods could change their performance. They developed a rig to measure the hysteresis parameters of the rods. After careful design, they used a bar magnet of 1 A·m^2 and eight hysteresis rods on both orthogonal axes to the bar magnet. The hysteresis rods had a square cross-section with dimensions 150mm × 1mm × 1mm and were composed of permalloy. After the rods were heat treated, measurements showed that their hysteresis parameters were well below the quoted hysteresis parameters for the material [63]. Unfortunately, due to the 2006 Dnepr

Figure 3.4: Artists conception of Munin, a Swedish small satellite which used PMAC [55].

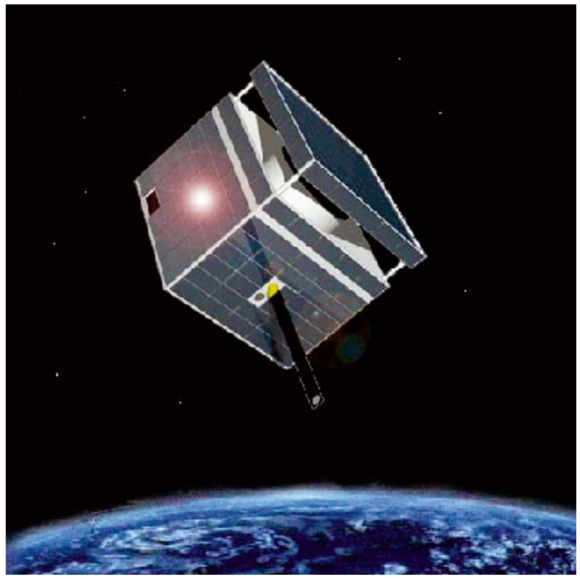
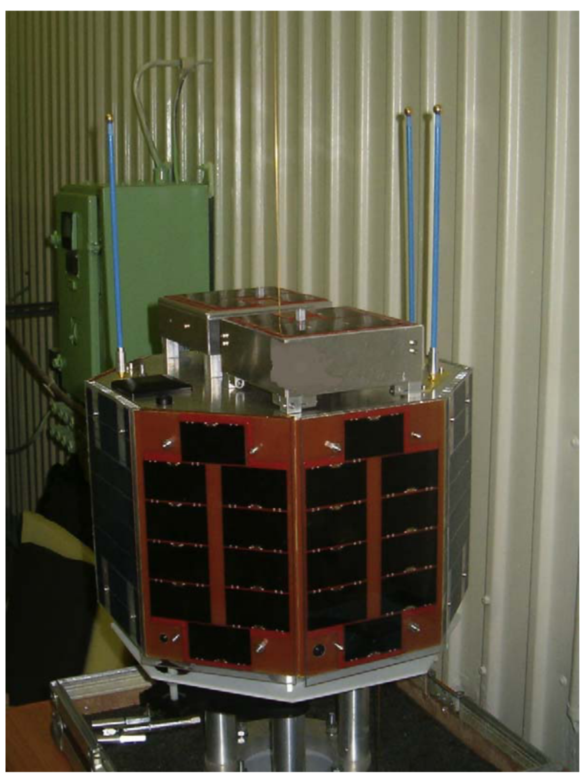


Figure 3.5: The UNISAT-4 satellite, a student satellite built by the University of Rome [63].



launch failure, UNISAT-4 did not achieve orbit [32].

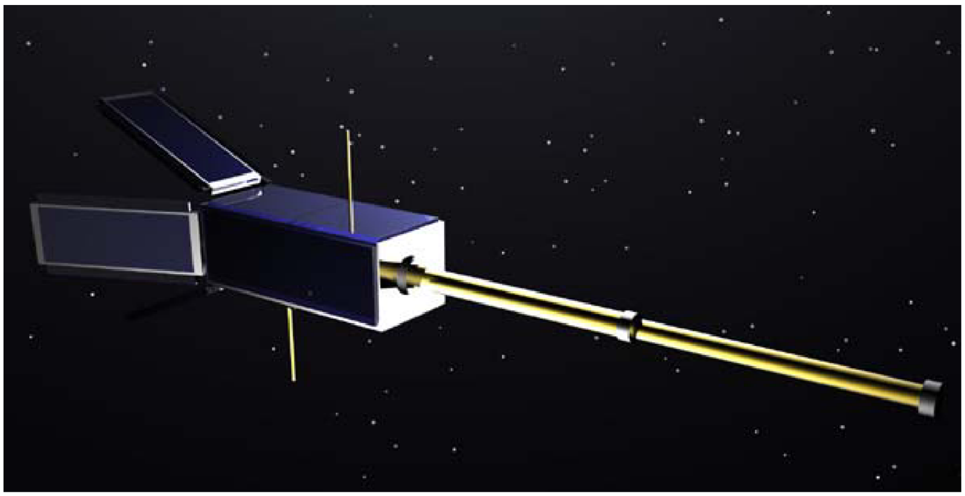
The next satellite from the University of Rome, EduSat, continued their work on PMAC development. Working with the University of Keldysh Institute of Applied Mathematics (KIAM), a new hysteresis parameter experimental set-up was developed. This set-up allows the hysteresis parameters to be measured along the length of the hysteresis rod. Researchers found that the maximum magnetic flux density was highest at the center of the hysteresis rod and decreased towards the ends [4]. The hysteresis rod measurements described in Section 7.3 use a sense coil with a length equivalent to the hysteresis rod length; this ensures that the average interior magnetic flux density of the rod is measured.

In 2001, the 23 kg Sapphire microsatellite was launched. Designed by Stanford university, it used a PMAC system to de-spin the satellite and ensure that an imaging sensor was pointed toward earth in the northern hemisphere. The communication antennas were also painted to impart a small radiation pressure torque which ensured a roll to prevent one side from always facing the sun (this attitude control is known as the "controlled tumble" and has been used for many AMSAT spacecraft). Sapphire was ejected from the launch vehicle with a tumble of multiple degrees per minute. This spin was reduced to 1.2 rpm about the major inertia axis with a few days due to the PMAC system. Radiation pressure caused the satellite to settle to 0.1 rpm [74].

3.1.3 CubeSats using Passive Magnetic Attitude Control

In summer of 2000, Bob Twiggs and researchers at Stanford university envisioned a new nanosatellite standard they called CubeSat [76]. This standard was soon accepted by universities across the country; PMAC was used as the stabilization method for many of these satellites. One of these early CubeSats that used PMAC was QuakeSat, built by Stanford university [52]. Launched in June 2003, QuakeSat relied upon solar panel currents and a single IR sensor to determine attitude. Unfortunately, the loss of a multiplexer early in the mission meant that the solar array currents were not available. QuakeSat used a 2.933 $A{\cdot}m^2$ bar magnet in combination with two 0.6cm $\times$ 1.2cm $\times$ 31cm rods of permalloy 49NM along the length of the satellite [69]. Using their single IR sensor, the QuakeSat team estimated that their satellite was generally nadir pointing, with a roll rate of once every 15 - 20 min [7].

Figure 3.6: Artist's conception of the QuakeSat CubeSat. Built by Standford University, QuakeSat used a PMAC system [52].

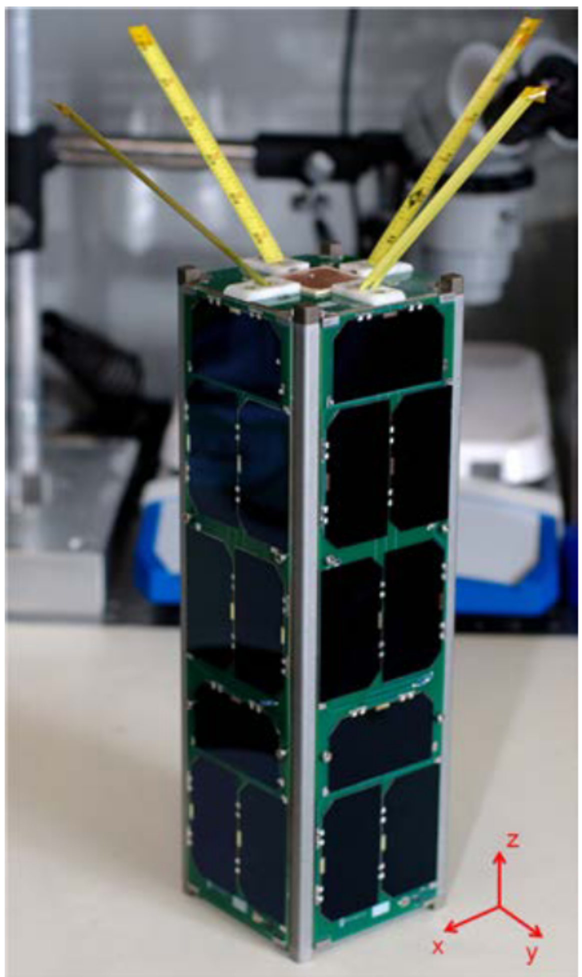


The Radio Aurora Explorer (RAX) was the first CubeSat funded by the National Science Foundation to study space weather [18]. Built by the University of Michigan, the RAX mission is actually composed of two satellites, RAX-1 and RAX-2; RAX-2 was launched to continue the science mission after the RAX-1 solar panels were found to be faulty [19].

The RAX team attempted to calculate the performance of its PMAC system using a dynamic model. Students at the University of Michigan developed the Lie Group Variational Integrator (LGVI). The LGVI is designed to model the rotation of a rigid body while conserving the constraints of the rotation matrix as well as the system energy [58]. More detail on this integrator is found in Section 8.1.7.2.

The RAX team used the LGVI to developed a simulation to predict the response of their CubeSat. However, they ignored the possibility of the satellite bar magnet saturating the hysteresis rods. To simplify integration, the team designed the PMAC system such that the bar magnet and hysteresis rods are on the same 10cm×10cm board which fits the form factor of other electronics

Figure 3.7: The RAX-1 3U CubeSat. Built by the University of Michigan, RAX-1 and RAX-2 used identical PMAC systems [19].



boards; this is not an optimal design as will be explained in Chapter 5. The team also used the closed-magnetic circuit hysteresis parameters as input to their numeric model; this results in a gross overestimate of the hysteresis dampening performance (see Section 7.3). These issues seriously degrade the predictive capability of the simulation.

The settling time of RAX-1 is not known as attitude data was only collected for three single-orbit periods, each 15 days apart [71]. However, RAX-2 attitude data shows that the satellite converges to within 20° of the local magnetic field two months after launch [73]. The predicted settling time of the RAX mission has not been published but personal communication with a member of the RAX team indicates the satellite was expected to align within days. This large discrepancy indicates that accurate PMAC dynamics simulation is both difficult and highly useful. Also, the RAX-2 satellite uses a bar magnet magnetic moment of 3.2 A·m^2 [73]; this powerful magnet (relative to a CubeSat inertia matrix) may have introduced the high initial rotation rates experienced by RAX-2 (see Section 5.2).

3.2 Analytical Models

An analytical model of PMAC is complicated by the use of hysteresis rods, whose torque depends on both the current orientation and the previous magnetism induced within the rods. Further complication is introduced when considering a real earth field, which is difficult to model. To combat this, the analytical models derived below simplify the hysteresis effect by assuming some average damping, usually in the form of an angular velocity coefficient. The earth's magnetic field is simplified with either the dipole assumption or an average magnetic field strength.

3.2.1 Fischell Analytical Model (1961)

Robert Fischell derived the first analytical model of PMAC for the Transit satellite program [25]. He starts with the assumption of a completely symmetric satellite ($I = I_{xx} = I_{yy} = I_{zz}$) and the magnitude of magnetic torque $\tau = M\mu_0 H_0 \sin\theta$ where M is the magnetic moment of the bar magnet, μ_0 is the permeability of free space, H_0 is the earth's local magnetizing field, and θ

is the angle between the bar magnet and the earth field direction. Fischell chooses to ignore the hysteresis torque at first in order to form Euler's rotational equations of motion for an undamped satellite axis:

$$I\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + M\mu_0 H_0 \sin\theta = 0. \tag{3.1}$$

Then, after making the small angle assumption for θ, the undamped angle relative to the magnetic field is given as $\theta_N = \theta_0 \cos(2\pi f t)$ where the natural frequency f is:

$$f = \frac{1}{2\pi}\sqrt{\frac{M\mu_0 H_0}{I}}. \tag{3.2}$$

Fischell, ignoring other disturbance torques, then defines the energy loss per time due to hysteresis cycling as:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{NV}{8\pi^2}\sqrt{\frac{M\mu_0 H_0}{I}}\oint H\mathrm{d}B \tag{3.3}$$

where V is the volume of the hysteresis material, N is the number of rods, and $\oint H\mathrm{d}B$ is the area of the hysteresis loop. Fischell obtained the hysteresis loop of the chosen rods experimentally and determined that the hysteresis loop area may be approximated as $\oint H\mathrm{d}B = \alpha H_m^3$ where H_m is the peak magnetizing field of the hysteresis loop and α is an empirically-derived constant. The peak magnetizing field for a rod perpendicular to the local earth field is given as $H_m = H_0 \sin\theta_m$ where θ_m is the max angular displacement between the bar magnet and the earth field.

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\frac{\alpha NV}{8\pi^2}\sqrt{\frac{M\mu_0}{I}}H_0^{7/2}\sin^3\theta_m. \tag{3.4}$$

Here Fischell chooses to define a constant $k = \frac{\alpha NV}{8\pi^2}\sqrt{\frac{M\mu_0}{I}}H_0^{7/2}$. Now, the potential energy of the bar magnet can be defined by integration of the magnitude of magnetic torque from equilibrium to the max displacement:

$$E(\theta) = \int_0^{\theta_m} M\mu_0 H_0 \sin\theta \mathrm{d}\theta = M\mu_0 H_0 \left(1 - \cos\theta_m\right). \tag{3.5}$$

Now, taking the derivative of Equation 3.5 with respect to time, combining with Equation 3.4, and separating variables yields:

$$-\frac{\mathrm{d}\theta_m}{\sin^2\theta_m} = \frac{k\mathrm{d}t}{M\mu_0 H_0} \tag{3.6}$$

which becomes the following after integration:

$$\cot\theta_m = \frac{k}{M\mu_0 H_0}t + C. \tag{3.7}$$

The constant can be solved for by defining an angle θ_0 to which the satellite is displaced at time $t = 0$. Thus, $C = \cot\theta_0$, and the following is the expression for the maximum angular displacement over time:

$$\theta_m(t) = \arccos\left(\frac{k}{M\mu_0 H_0}t + \cot(\theta_0)\right). \tag{3.8}$$

Finally, Fischell combines the undamped angle with the maximum damped angle to yield the angle of displacement from the magnetic field with respect to time:

$$\theta(t) = \arccos\left(\frac{k}{M\mu_0 H_0}t + \cot(\theta_0)\right)\cos 2\pi f t. \tag{3.9}$$

The settling time is easily found by modifying Equation 3.8:

$$t_{\text{settle}} = \frac{M\mu_0 H_0}{k}\left(\cot\theta_f - \cot\theta_0\right) \tag{3.10}$$

where θ_f is the final angular displacement. To recap, Fischell has defined an analytical solution with the following assumptions:

- Entirely symmetric satellite
- Small angle between B-field & bar magnet axis
- Orbit-average magnetic field strength used instead of position-dependent vector
- No other (non-magnetic) disturbance torques
- Cubic approximation of hysteresis area

The above assumptions are quite limiting. Even with a symmetric satellite, this analytical model is ineffective for the beginning of dampening, where high angular velocities are typical.

3.2.2 Mesch et al. Analytical Model (1966)

Mesch et al. developed a more comprehensive analytical model [22]. They start with Equation 2.3, then define the angular velocities in terms of Euler angle rates. However, here they make the assumption that the Euler angles are always a small angle for a satellite with attitude control. A unique aspect of this model is the development of the equations of motion in terms of the orbit true anomaly instead of time, allowing a dipole magnetic field to be included within the model. However, there are many assumptions made:

- Angle between B-field & bar magnet is a small angle
- Polar orbit (inclination = 90°)
- Dipole magnetic field
- Dampening torque is in constant proportion to angular velocity

Due to these assumptions, this model was generally used to determine the periodic motion of a satellite after it had settled to oscillating about the magnetic field.

3.2.3 Kammüller Analytical Model (1971)

Kammüller takes a different approach [39], [40]. Rather than starting from Euler's rotational equations of motion (Equation 2.3), the Lagrangian is calculated assuming a 3-1-3 Euler angle set. By using the Lagrangian equations of motion, Kammüller is able to account for the gyroscopic torques of the spacecraft without solving six coupled equations of motion. For the Lagrangian, the potential energy is defined as Equation 3.5, but with a transformation used to convert θ_m to a function of Euler angles and the magnetic declination. A series expansion accounts for the magnetic field strength and the magnetic declination as a function of the orbital frequency and time. Here Kammüller introduces a "slow" time variable $\tau = \omega_e t$ where ω_e is the earth's rotation rate. This assumption, based on the significant difference between the earth's rotation rate and the orbital frequency, allows Kammüller to treat some timescales as constant with respect to the "fast" time variable t.

Kammüller shows that, for near-polar orbits (i$\approx 90°$), there exists a roll resonance for specific values of $\Delta = (I_{xx} - I_{yy})/I_{zz}$, where I_{xx} is the maximum moment of inertia, and I_{zz} is the minimum. He points out that there are three possible solutions to the roll equation (after transients have been damped): a) nonresonant rest-position, b) resonant oscillation, and c) resonant rotational solutions. The roll resonance is due to coupling between pitch and roll motions of the spacecraft, and thus changes depending on the spacecraft Δ. By changing the spacecraft Δ, the desired stability (a, b, or c) may be set.

Kammüller makes the following assumptions in his analysis:

- Circular polar orbit
- Dipole Earth field
- Pure pitch (major inertia axis) motion while following magnetic field lines
- "Slow" time used to consider diurnal rotation negligible with respect to orbital motion
- Hysteresis dampening described by matrix of dampening coefficients multiplied by Euler angles and Euler angle rates

Other analytical PMAC models could not be found in the literature. To date, none of the analytical models have solved for the settling time of a non-symmetric satellite.

3.3 Numerical Simulations

Numerical models have the advantage of not making the simplifying assumptions of the analytic models. As a result, numerical models have the potential to accurately predict the full dynamics of the system. However, numerical models have their own disadvantages; a balance must be sought between simulation accuracy and computational cost. Also, the model itself can introduce errors if not properly defined. Numerical models which have been used in the past are presented below.

3.3.1 Chen (1965)

Chen [15] uses a 3-1-3 Euler angle set to describe the rotation of the body frame with respect to the inertial frame. An inclined dipole is used to model the earth field. An interesting note is the inclusion of another dampening torque. This "shorted coil" dampening torque is due to closed windings about each hysteresis rod which have current induced within them due to the earth field. This current, in turn, torques the satellite according to Equation 2.6. The hysteresis loop is modeled as a parallelogram, which does not account for minor hysteresis loops which occur as the satellite starts to track the earth field. After the torques are defined, Equation 2.3 is used to define the equations of motion. In order to avoid the singularity associated with Euler angles, quaternions are used to model the attitude during integration.

Chen uses the following assumptions:

- Dipole Earth field
- "Shorted Coil" dampening in addition to hysteresis rod dampening
- Parallelogram hysteresis loop

3.3.2 Ovchinnikov & Penkov (2002) - Munin

Ovchinnikov & Penkov investigate the motion of a 6 kg axisymmetric satellite with a bar magnet and hysteresis dampening [57]. First, a magnetic frame is defined by the direction of the local earth field and the orbit plane of the satellite. The earth field is modeled as a dipole, and a parallelogram model is used for the hysteresis loop. Equation 2.1 serves as the equation of motion. A 2-1-3 Euler angle is used as attitude parameters. The equations of motion are then written in dimensionless form and a few key assumptions are made: a strong bar magnet dominates the external torques and the initial angular velocity of the satellite about the symmetry axis is roughly equivalent to the mean motion of the satellite. These assumptions allow the average equations of motion to be developed. These average equations of motion are investigated with numeric analysis.

Ovchinnikov & Penkov use the following assumptions:

- Strong bar magnet
- "Improved" Parallelogram model (able to generate minor loops near origin only)
- Averaged equations of motion

3.3.3 CUBESIM (2004) and SNAP (2009)

Both CUBESIM and the Smart Nano-satellite Attitude Propagator (SNAP) are PMAC attitude simulations developed by students (Levesque [47] and Rawashdeh [61], respectively) in an attempt to simulate the response of a PMAC system for satellites they were working on at the time. Both models use the Matlab Simulink environment along with the Dormand-Prince 45 variational numeric integrator (generally known as the DOPRI method, known in MATLAB as `ode45`), and both models use the parallelogram model to determine the hysteresis torque given the magnetic field input. Both models include the effect of gravity gradient torque, but ignore the other environmental torques (drag, solar pressure, magnetic residual, eddy current). Finally, both models use the closed magnetic circuit hysteresis parameters to form their parallelogram models. In both models, these assumptions and incorrect inputs add up to a simulation that converges very quickly (when CSSWE initial conditions are input, CUBESIM converges to the local magnetic field within a few orbits). Also, the CUBESIM output was not found to converge as lower time steps were used. Frustrations with CUBESIM inconsistencies was one of the motivations for work on a new PMAC simulation.

CUBESIM and SNAP use the following assumptions:

- Parallelogram hysteresis loop (no minor loops)
- Runge-Kutta45 numeric integrator

- No environmental torques included except gravity gradient
- Closed magnetic circuit hysteresis parameters

3.3.4 Park et al. (2010) & Lee et al. (2011) - RAX

The RAX CubeSat team recognized the faults with previous PMAC simulation tools [47]. They developed their own numeric integration model based on Equation 2.3. The RAX model includes magnetic and hysteresis torques, and rotation matrices are the chosen attitude coordinates. The RAX team uses the Lie Group Variational Integrator (LGVI) developed by Lee [45] at the University of Michigan. This energy-conserving numeric integrator is used in an attempt to limit system energy change due to the numeric integrator itself; the LGVI is described in Section 8.1.7 and tested versus other integrators in Section 8.3.

The RAX simulation uses the Flatley empirically-derived hysteresis loop model (described in Section 8.1.6.5). Unfortunately, the RAX simulation incorrectly uses closed magnetic circuit hysteresis parameters. Although students at the University of Michigan developed this PMAC simulator, only preliminary applications have been performed [58]. To date, the RAX team has not validated the performance of their simulation.

3.4 Hysteresis Measurement to Date

Only one other small satellite research group has made a priority of hysteresis measurement. Most teams incorrectly assume that the closed magnetic circuit hysteresis parameters of the hysteresis rod material may be used for simulation purposes, ([47],[61],[58]) but as is shown in Section 7.3, this is not the case. The very first PMAC mission, Transit 1B, measured the area of its hysteresis loops and used it as an input to its analytical model [25]. Of the modern missions, only the University of Rome (UNISAT-4, EduSat) have measured their rod hysteresis loops.

The UNISAT-4 team measured the hysteresis rod by placing a magnetometer close to one end of the rod and measuring the B-field resultant of magnetization changes within the rod [63].

However, their sense coil was not surrounding the hysteresis rod, they required the use of a scaling factor to translate from measured data away from the rod to some internal average B-field within the rod. Also, their measurement technique lacks the ability to perform system measurements. However, they did achieve measured hysteresis loops within an order of magnitude of those presented in Section 7.3.

Recently, the University of Rome built a new measurement system which uses a forcing coil and sense coil to determine the hysteresis parameters as a function of rod length (because the sense coil is much shorter than the hysteresis rod). The results from this measurement system yield hysteresis parameters ($H_c = 1.135$ A/m, $B_r = 0.0073$ Tesla, $B_s = 0.1315$ Tesla, Area=0.4263 J$\cdot$m^{-3} for a 200 mm $\times$ 1 mm $\times$ 1 mm HyMu-80 rod [5]) which are vastly different from the closed magnetic circuit material hysteresis parameters (see Table 7.2). Other work measuring PMAC hysteresis rods is an analysis of various magnetic materials which could be used to fabricate hysteresis rods [24].